

M2 JAN 11

1. A cyclist starts from rest and moves along a straight horizontal road. The combined mass of the cyclist and his cycle is 120 kg. The resistance to motion is modelled as a constant force of magnitude 32 N. The rate at which the cyclist works is 384 W. The cyclist accelerates until he reaches a constant speed of  $v$  m s<sup>-1</sup>.

Find

- (a) the value of  $v$ , (3)
- (b) the acceleration of the cyclist at the instant when the speed is 9 m s<sup>-1</sup>. (3)

$$32 \leftarrow \boxed{120} \rightarrow \frac{384}{v}$$

$$a) \frac{384}{v} = 32 \Rightarrow v = \underline{12 \text{ m s}^{-1}}$$

$$b) \frac{384}{9} - 32 = 120a \Rightarrow a = \underline{\underline{\frac{4}{45} \text{ m s}^{-2}}}$$

2. A particle of mass 2 kg is moving with velocity  $(5\mathbf{i} + \mathbf{j})$  m s<sup>-1</sup> when it receives an impulse of  $(-6\mathbf{i} + 8\mathbf{j})$  N s. Find the kinetic energy of the particle immediately after receiving the impulse. (5)

$$\text{Initial Mom} = 10\mathbf{i} + 2\mathbf{j}$$

$$\text{Impulse} = -6\mathbf{i} + 8\mathbf{j}$$

$$\text{final Mom} = 4\mathbf{i} + 10\mathbf{j} \Rightarrow v = 2\mathbf{i} + 5\mathbf{j}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(2)(2^2 + 5^2) = \underline{\underline{29 \text{ J}}}$$

3. A particle moves along the  $x$ -axis. At time  $t = 0$  the particle passes through the origin with speed 8 m s<sup>-1</sup> in the positive  $x$ -direction. The acceleration of the particle at time  $t$  seconds,  $t \geq 0$ , is  $(4t^3 - 12t)$  m s<sup>-2</sup> in the positive  $x$ -direction.

Find

- (a) the velocity of the particle at time  $t$  seconds, (3)
- (b) the displacement of the particle from the origin at time  $t$  seconds, (2)
- (c) the values of  $t$  at which the particle is instantaneously at rest. (3)

$$a) a = 4t^3 - 12t$$

$$v = t^4 - 6t^2 + C, t=0, v=8 \Rightarrow C=8$$

$$v = t^4 - 6t^2 + 8$$

$$b) s = \frac{1}{5}t^5 - 2t^3 + 8t + C \quad s=0, t=0 \Rightarrow C=0$$

$$s = \frac{1}{5}t^5 - 2t^3 + 8t$$

$$c) v=0 \quad t^4 - 6t^2 + 8 = 0$$

$$(t^2 - 4)(t^2 - 2) = 0$$

$$t = \sqrt{4} = \underline{2} \quad t = \sqrt{2} = \underline{1.41 \text{ sec}}$$







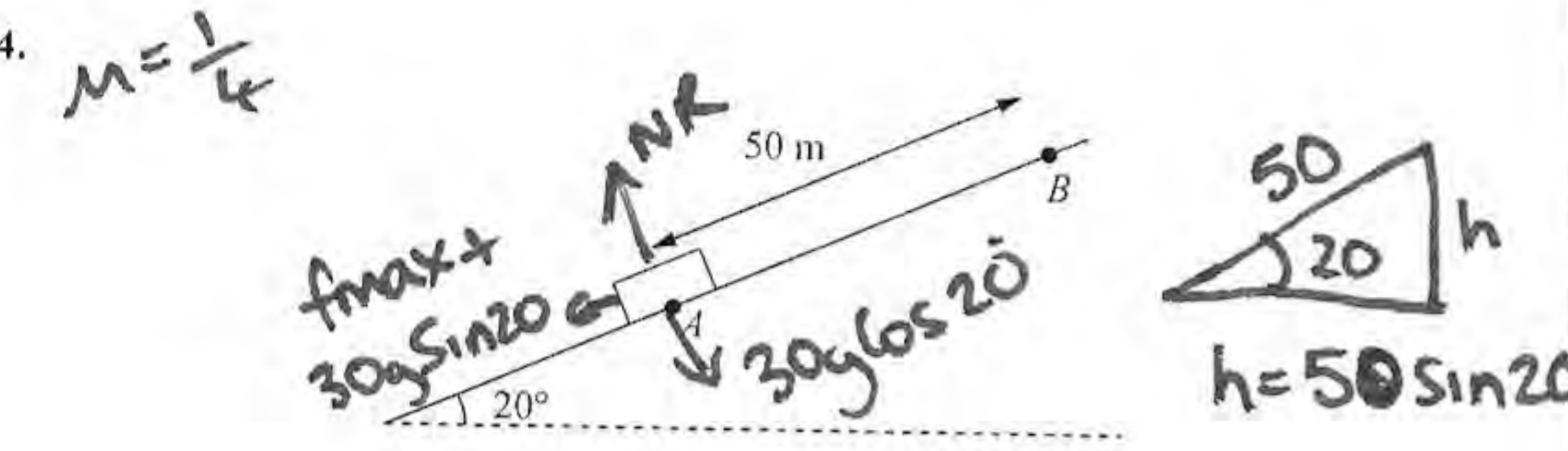


Figure 1

A box of mass 30 kg is held at rest at point  $A$  on a rough inclined plane. The plane is inclined at  $20^\circ$  to the horizontal. Point  $B$  is 50 m from  $A$  up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from  $A$  to  $B$  by a force acting parallel to  $AB$  and then held at rest at  $B$ . The coefficient of friction between the box and the plane is  $\frac{1}{4}$ . Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(a) find the work done in dragging the box from  $A$  to  $B$ .

The box is released from rest at the point  $B$  and slides down the slope. Using the work-energy principle, or otherwise,

(b) find the speed of the box as it reaches  $A$ .

a)  $f_{max} = \mu NR = \frac{1}{4} (30g \cos 20)$   
 $f_{max} = 69.067 \dots$

Wd against friction =  $69.067 \times 50$

$Wd = 169.62 \times 50 = 8481.1 \text{ J}$

b)  $KE_A = PE_B - WD_{AB}$

$\frac{1}{2}mv^2 = 30g \times 50 \sin 20 - 3453.4$

$15v^2 = 1574.3 \dots$        $v = 10.24$   
 $v = 10.2 \text{ ms}^{-1}$

The uniform L-shaped lamina  $ABCDEF$ , shown in Figure 2, has sides  $AB$  and  $FE$  parallel, and sides  $BC$  and  $ED$  parallel. The pairs of parallel sides are 9 cm apart. The points  $A, F, D$  and  $C$  lie on a straight line.

$AB = BC = 36 \text{ cm}$ ,  $FE = ED = 18 \text{ cm}$ .  $\angle ABC = \angle FED = 90^\circ$ , and  $\angle BCD = \angle EDF = \angle EFD = \angle BAC = 45^\circ$ .

(a) Find the distance of the centre of mass of the lamina from

- (i) side  $AB$ ,
- (ii) side  $BC$ .

The lamina is freely suspended from  $A$  and hangs in equilibrium.

(b) Find, to the nearest degree, the size of the angle between  $AB$  and the vertical.

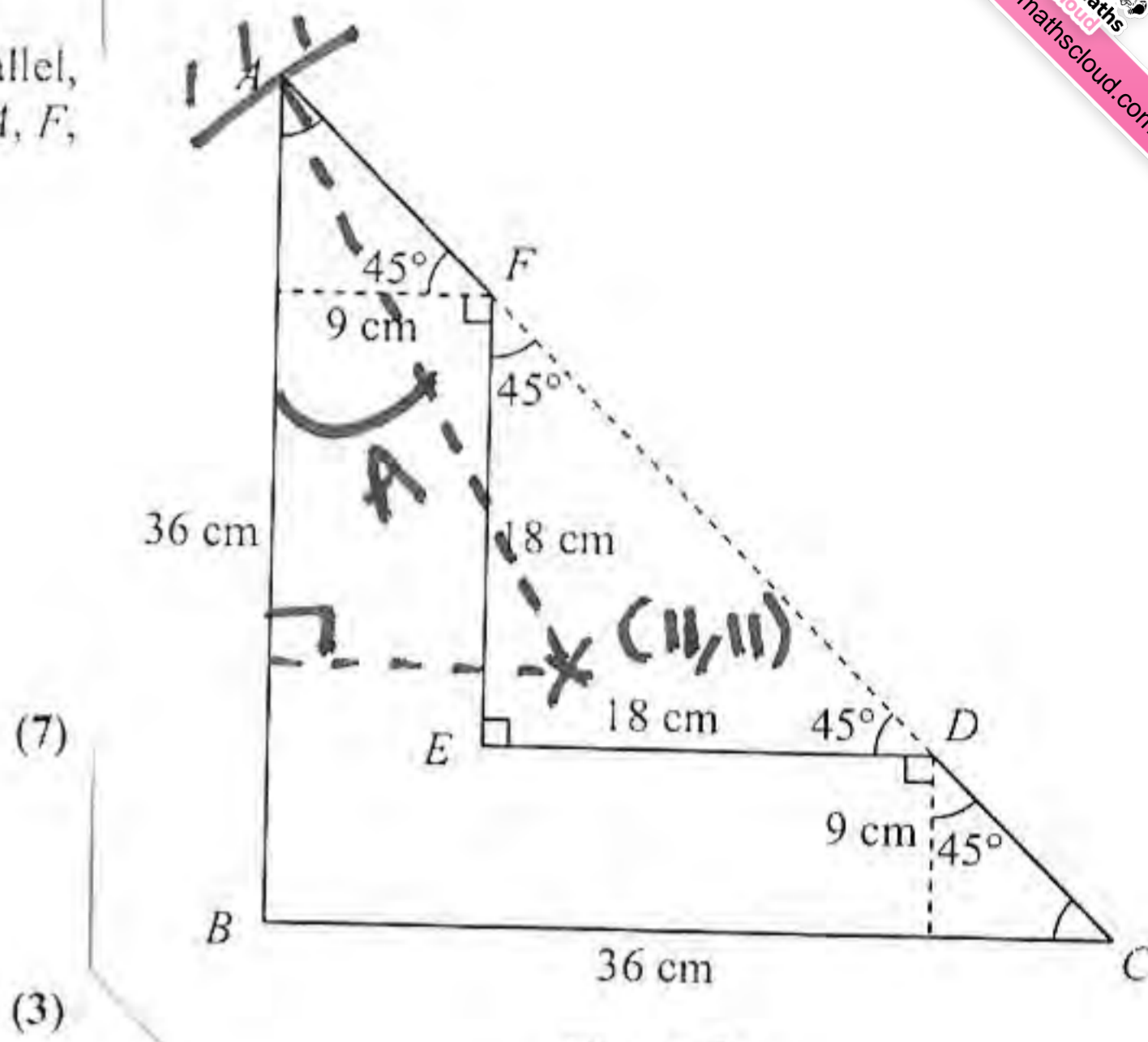
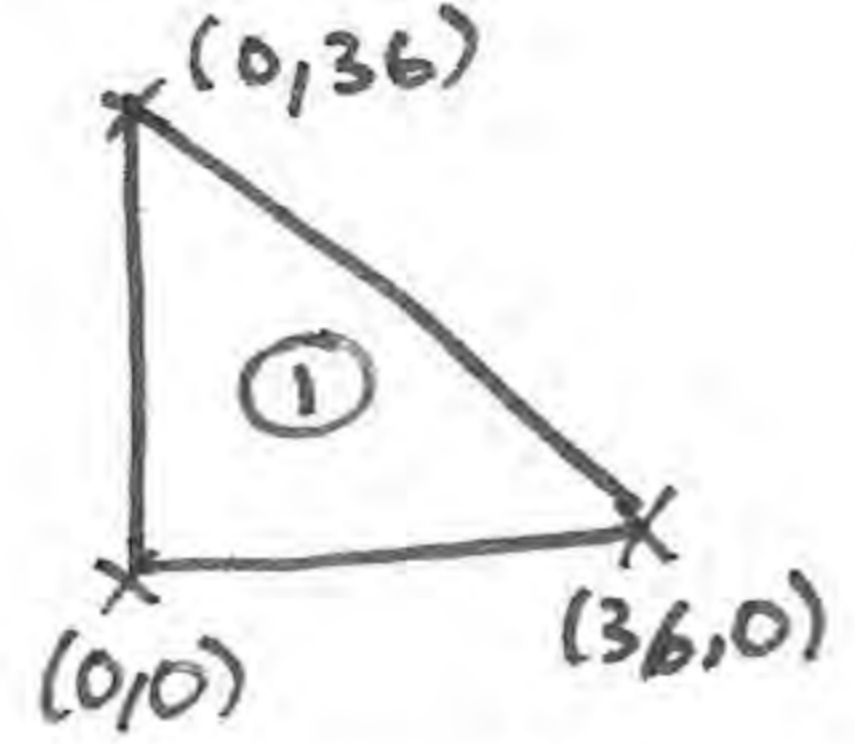


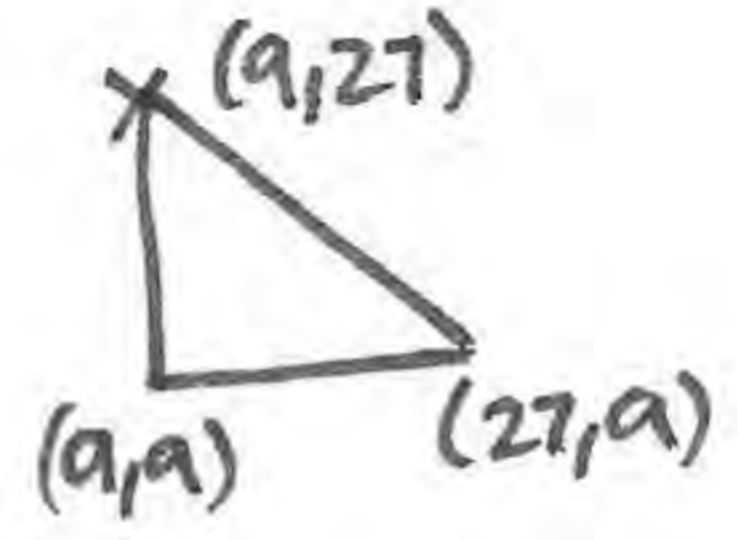
Figure 2



$\bar{x}_1 = \frac{0+0+36}{3} = 12$

$A_1 = \frac{36^2}{2} = 648$

$648 \text{ kg} \times 12 + 162 \text{ kg} \times 15 = 486 \text{ kg} \times \bar{x}$



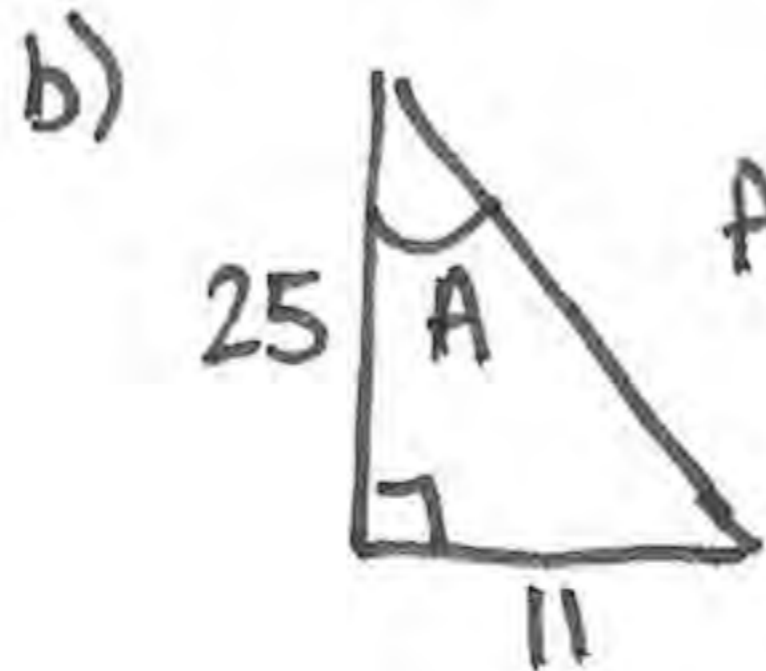
$\bar{x}_2 = \frac{9+9+27}{3} = 15$

$A_2 = \frac{18^2}{2} = 162$

= Total

Total =  $486 (\bar{x}, \bar{y})$

$\Rightarrow \bar{x} = 11 \text{ cm}$  (ii)  $11 \text{ cm}$



$A = \tan^{-1}(\frac{11}{25}) = 23.7^\circ$

$A = 24^\circ$



6. [In this question, the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are in a vertical plane,  $\mathbf{i}$  being horizontal and  $\mathbf{j}$  being vertically upwards.]

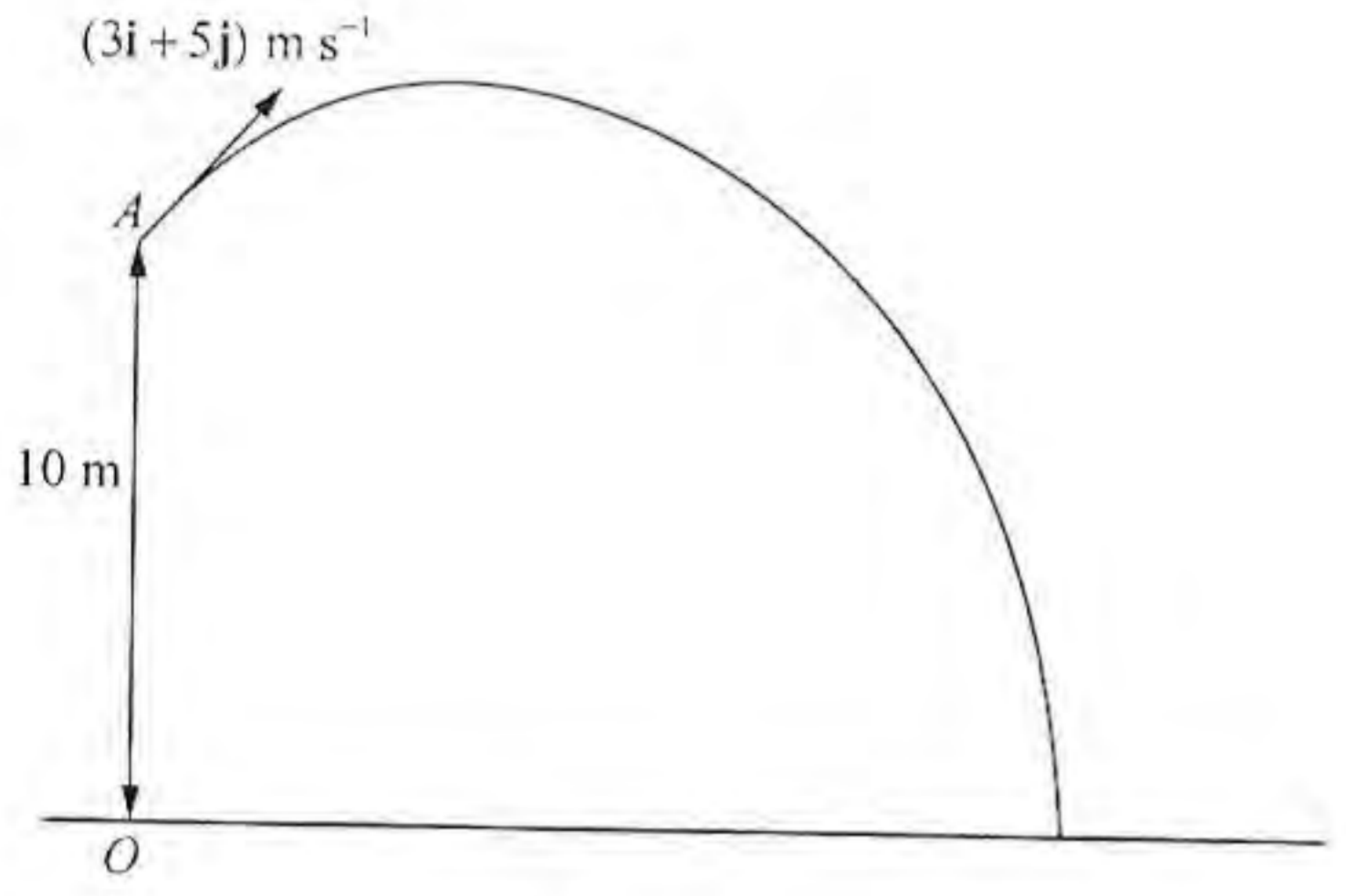


Figure 3

At time  $t = 0$ , a particle  $P$  is projected from the point  $A$  which has position vector  $10\mathbf{j}$  metres with respect to a fixed origin  $O$  at ground level. The ground is horizontal. The velocity of projection of  $P$  is  $(3\mathbf{i} + 5\mathbf{j}) \text{ m s}^{-1}$ , as shown in Figure 3. The particle moves freely under gravity and reaches the ground after  $T$  seconds.

(a) For  $0 \leq t \leq T$ , show that, with respect to  $O$ , the position vector,  $\mathbf{r}$  metres, of  $P$  at time  $t$  seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j} \quad (3)$$

(b) Find the value of  $T$ . (3)

(c) Find the velocity of  $P$  at time  $t$  seconds ( $0 \leq t \leq T$ ). (2)

When  $P$  is at the point  $B$ , the direction of motion of  $P$  is  $45^\circ$  below the horizontal.

(d) Find the time taken for  $P$  to move from  $A$  to  $B$ . (2)

(e) Find the speed of  $P$  as it passes through  $B$ . (2)

a)  $\vec{H} \quad v = 3 \quad s = 3t$

$v \uparrow \quad u \uparrow = 5 \quad y = 5t - \frac{1}{2}(9.8)t^2$

$a \uparrow = -9.8 \quad y = 5t - 4.9t^2$

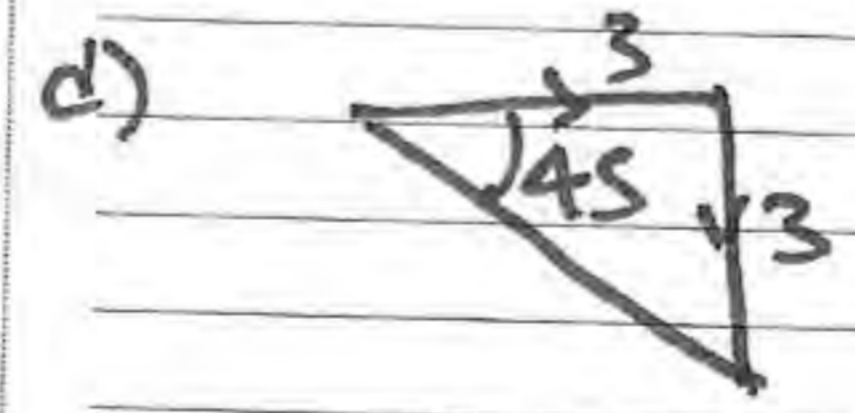
$s = y \quad \text{from ground} = 10 + 5t - 4.9t^2$

$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$

b)  $t = T \quad \mathbf{j} = 0 \quad 4.9t^2 - 5t - 10 = 0$

$t = \frac{5 + \sqrt{5^2 - 4(4.9)(-10)}}{9.8} \quad t = \underline{2.03 \text{ sec}}$

c)  $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$



$5 - 9.8t = 3 \quad t = \frac{2}{9.8} = \underline{0.204}$

e)  $\sqrt{3^2 + 3^2} = 3\sqrt{2} = \underline{4.24 \text{ ms}^{-1}}$



7.

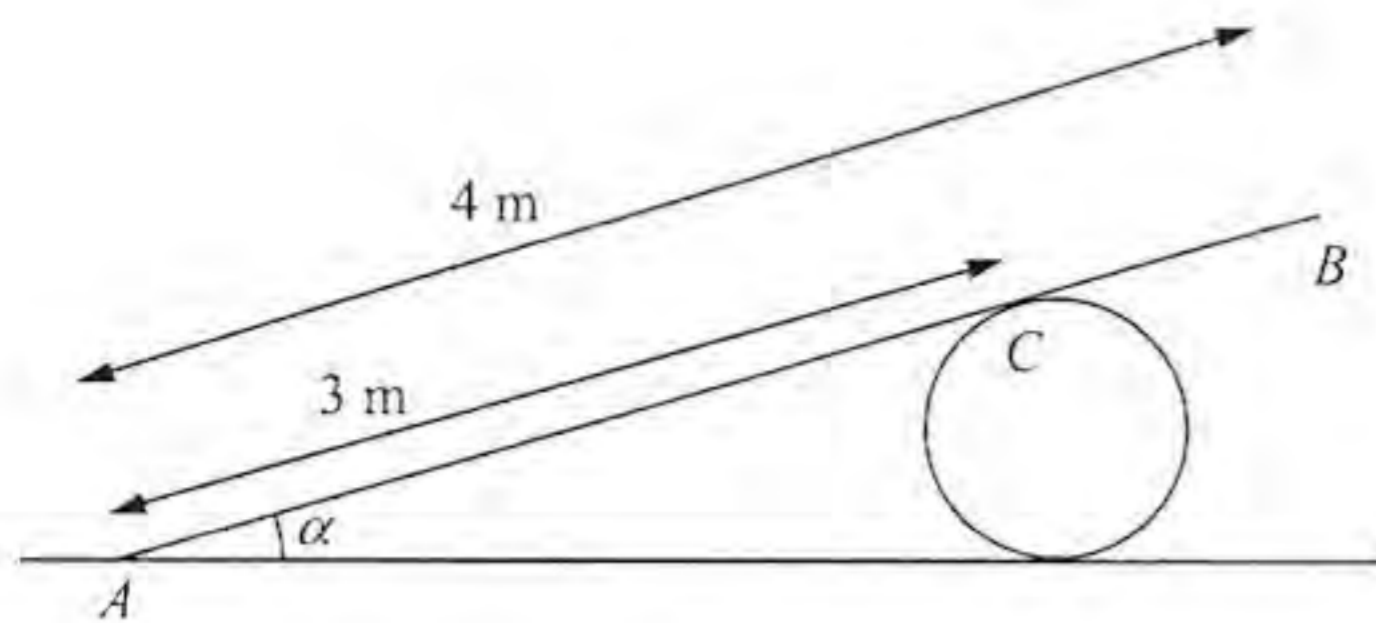
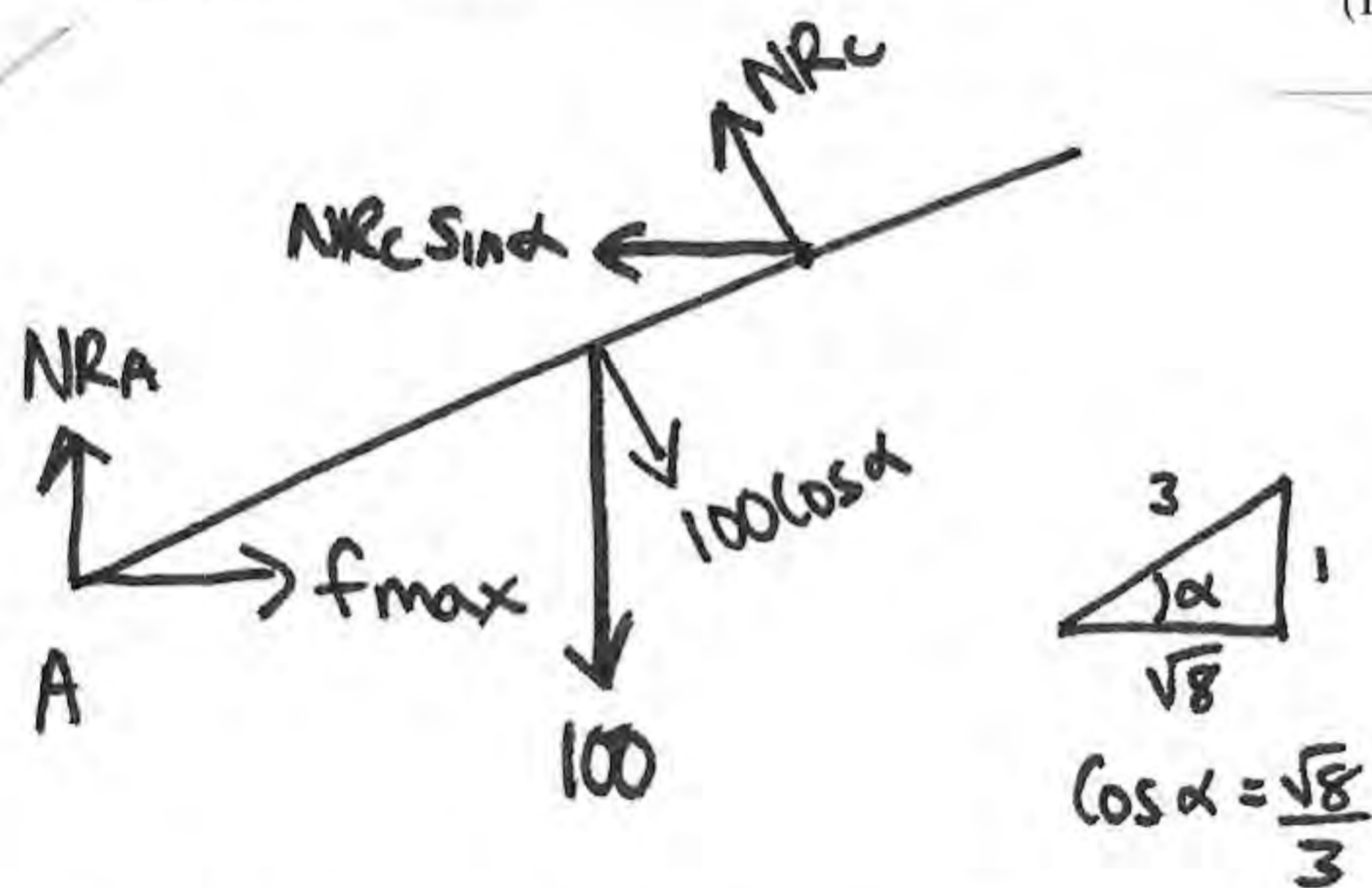


Figure 4

A uniform plank  $AB$ , of weight  $100\text{ N}$  and length  $4\text{ m}$ , rests in equilibrium with the end  $A$  on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is  $C$ , where  $AC = 3\text{ m}$ , as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{1}{3}$ . The coefficient of friction between the plank and the ground is  $\mu$ . Modelling the plank as a rod, find the least possible value of  $\mu$ .

(10)



$$\text{A} \downarrow \quad 100 \times \frac{\sqrt{8}}{3} \times 2 = 3 NR_c \Rightarrow NR_c = 62.85 \dots$$

$$\rightarrow = \leftarrow \quad NR_c \sin \alpha = f_{\max} \Rightarrow f_{\max} = 20.95 \dots$$

$$\uparrow = \downarrow \quad NR_A + NR_c \cos \alpha = 100 \Rightarrow NR_A = 40.74 \dots$$

$$f_{\max} = \mu \times NR_A \Rightarrow 20.95 \dots = 40.74 \dots \mu$$

$$\mu = 0.514$$

8. A particle  $P$  of mass  $m\text{ kg}$  is moving with speed  $6\text{ m s}^{-1}$  in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is  $64\text{ J}$ . The coefficient of restitution between  $P$  and the wall is  $\frac{1}{3}$ .

(a) Show that  $m = 4$ .

(6)

After rebounding from the wall,  $P$  collides directly with a particle  $Q$  which is moving towards  $P$  with speed  $3\text{ m s}^{-1}$ . The mass of  $Q$  is  $2\text{ kg}$  and the coefficient of restitution between  $P$  and  $Q$  is  $\frac{1}{3}$ .

(b) Show that there will be a second collision between  $P$  and the wall.

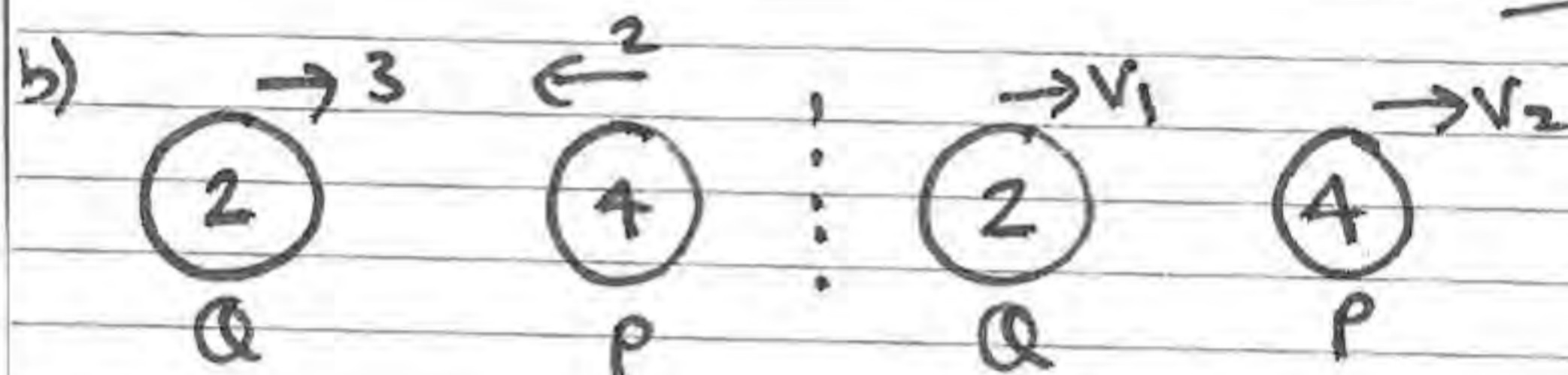
(7)

$$\text{a) } e = \frac{v_{\text{sep}}}{v_{\text{app}}} = \frac{v}{6} = \frac{1}{3} \Rightarrow v = 2\text{ m s}^{-1}$$

$$KE_{\text{before}} = \frac{1}{2} m v^2 = 18m$$

$$KE_{\text{after}} = \frac{1}{2} m v^2 = 2m \Rightarrow \text{loss} = 16m = 64$$

$$m = 4\text{ kg}$$



$$2 \times 3 + 4 \times (-2) = 2v_1 + 4v_2$$

$$-2 = 2v_1 + 4v_2 \quad (\times 3)$$

$$\frac{v_2 - v_1}{5} = \frac{1}{3} \quad -3v_1 + 3v_2 = 5 \quad (\times 2)$$

$$6v_1 + 12v_2 = -6 \quad v_2 = \frac{2}{9} \quad v_1 = \frac{-13}{9}$$

$$-6v_1 + 6v_2 = 10$$

$$18v_2 = 4$$

$$v_2 > 0, \text{ will hit wall}$$